

Astrophysical black holes as natural laboratories for fundamental physics and strong-field gravity

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Abstract Astrophysical tests of general relativity belong to two categories: 1) “internal”, i.e. consistency tests *within* the theory (for example, tests that astrophysical black holes are indeed described by the Kerr solution and its perturbations), or 2) “external”, i.e. tests of the many proposed extensions of the theory. I review some ways in which astrophysical black holes can be used as natural laboratories for both “internal” and “external” tests of general relativity. The examples provided here (ringdown tests of the black hole “no-hair” theorem, bosonic superradiant instabilities in rotating black holes and gravitational-wave tests of massive scalar-tensor theories) are shamelessly biased towards recent research by myself and my collaborators. Hopefully this colloquial introduction aimed mainly at astrophysicists will convince skeptics (if there are any) that space-based detectors will be crucial to study fundamental physics through gravitational-wave observations.

Keywords General Relativity · Black Holes · Gravitational Radiation

1 Introduction

The foundations of Einstein’s general relativity (GR) are very well tested in the regime of weak gravitational fields, small spacetime curvature and small velocities [1]. It is generally believed, on both theoretical and observational grounds (the most notable observational motivation being the dark energy problem), that Einstein’s theory will require some modification or extension at high energies and strong gravitational fields, and these modifications generally require the introduction of additional degrees of freedom in the theory [2].

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Because GR is compatible with all observational tests in weak-gravity conditions, a major goal of present and future experiments is to probe astrophysical systems where gravity is, in some sense, strong. The strength of gravity can be measured either in terms of the gravitational field $\varphi \sim M/r$, where M is the mass and r the size of the system in question¹, or in terms of the curvature [3]. A quantitative measure of curvature are tidal forces, related to the components $R^r_{0r0} \sim M/r^3$ of the Riemann tensor associated to the spacetime metric g_{ab} [4]. The field strength is related to typical velocities of the system by the virial theorem ($v \sim \varphi^{1/2} \sim \sqrt{M/r}$) so it is essentially equivalent to the post-Newtonian small velocity parameter v (or v/c in “standard” units). One could argue that “strong curvature” is in some ways more fundamental than “strong field”, because Einstein’s equations relate the stress-energy content of the spacetime to its curvature (so that “curvature is energy”) and because the curvature (not the field strength) enters the Lagrangian density in the action principle defining the theory: cf. e.g. Eq. (1) below.

It is perhaps underappreciated that in astrophysical systems one can “probe strong gravity” by observations of weak gravitational fields, and vice versa, observations in the strong-field regime may not be able to tell the difference between GR and its alternatives or extensions.

The possibility to probe strong-field effects using weak-field binary dynamics is nicely illustrated by the “spontaneous scalarization” phenomenon discovered by Damour and Esposito-Far  se [5]. The idea is that the coupling of the scalar with matter can allow some scalar-tensor theories to pass all weak-field tests, while at the same time introducing macroscopically (and observationally) significant modifications in the structure of neutron stars (NSs). If spontaneous scalarization occurs², the masses of the two stars in a binary can in principle be very different from their GR values. Therefore the dynamics of NS binaries will be significantly modified even when the binary members are sufficiently far apart that $v \sim \sqrt{M/r} \ll 1$. For this reason, “weak-field” observations of binary pulsars can strongly constrain a strong-field phenomenon such as spontaneous scalarization [12].

On the other hand, measurements of gas or particle dynamics in strong-field regions around the “extremely relativistic” Kerr black hole (BH) spacetime are *not* necessarily smoking guns of hypothetical modifications to general relativity. The reason is that classic theorems in Brans-Dicke theory [15,16,17], recently extended to generic scalar-tensor theories and $f(R)$ theories [18,19], show that solutions of the field equations in vacuum always include the Kerr metric as a special case. The main reason is that many generalizations of GR admit the vacuum equations of GR itself as a special case. This conclusion may be violated e.g. in the presence of time-varying boundary conditions, that

¹ Throughout this paper I will use geometrical units $G = c = 1$.

² The astrophysical plausibility of spontaneous scalarization is supported by detailed studies of stellar structure [6,7], numerical simulations of collapse [8,9,10] and stability analyses [11]. While the strength of spontaneous scalarization phenomena is already strongly constrained by observations of binary pulsars [12], semiclassical vacuum instabilities seem to offer a viable mechanism to “seed” nonzero scalar fields in stars [13,14].

could produce “BH hair growth” on cosmological timescales [20] and dynamical horizons [21].

The Kerr solution is so ubiquitous that probes of the Kerr *metric* alone will not tell us whether the correct theory of gravity is indeed GR. However, the *dynamics* of BHs (as manifested in their behavior when they merge or are perturbed by external agents [22]) will be very different in GR and in alternative theories. In this sense, gravitational radiation (which bears the imprint of the dynamics of the gravitational field) has the potential to tell GR from its alternatives or extensions.

To wrap up this introduction: our best bet to probe strong-field dynamics are certainly BHs and NSs, astronomical objects for which both $\varphi \sim M/r$ and the curvature $\sim M/r^3$ are large. However: 1) there is the definite possibility that weak-field observations may probe strong gravity, as illustrated e.g. by the spontaneous scalarization phenomenon; and 2) measurements of the metric around BH spacetimes will not be sufficient to probe GR, but *dynamical* measurements of binary inspiral and merger dynamics will be sensitive to the dynamics of the theory.

2 Finding contenders to general relativity

Let us focus for the moment on “external” tests, i.e. test of GR versus alternative theories of gravity. What extensions of GR can be considered serious contenders? A “serious” contender (in this author’s opinion) should at the very least be well defined in a mathematical sense, e.g. by having a well posed initial-value problem. From a phenomenological point of view, the theory must also be simple enough to make physical predictions that can be validated by experiments (it is perhaps a sad reflection on the current state of theoretical physics that one should make such a requirement explicit!).

An elegant and comprehensive overview of theories that have been studied in the context of space-based gravitational-wave (GW) astronomy is presented in [23]. Here I focus on a special subclass of extensions of GR whose implications in the context of Solar-System tests, stellar structure and GW astronomy have been explored in some detail. I will give a “minimal” discussion of these theories, with the main goal of justifying the choice of massive scalar-tensor theories as a particularly simple and interesting phenomenological playground.

Among the several proposed extensions of GR (see e.g. [2] for an excellent review), theories that can be summarized via the Lagrangian density

$$\begin{aligned} \mathcal{L} = & f_0(\phi)R \\ & - \varpi(\phi)g^{ab}\partial_a\phi\partial_b\phi - M(\phi) + \mathcal{L}_{\text{mat}}[\Psi, A^2(\phi)g_{ab}] \\ & + f_1(\phi)\mathcal{R}_{\text{GB}}^2 + f_2(\phi)R_{abcd}{}^*R^{abcd} \end{aligned} \quad (1)$$

have rather well understood observational implications for cosmology, Solar System experiments, the structure of compact stars and gravitational radiation from binary systems.

In the Lagrangian given above ϕ is a scalar-field degree of freedom (not to be confused with the gravitational field strength φ introduced earlier); R_{abcd} is the Riemann tensor, R_{ab} the Ricci tensor and R the Ricci scalar corresponding to the metric g_{ab} ; Ψ denotes additional matter fields. The functions $f_i(\phi)$ ($i = 0, 1, 2$), $M(\phi)$ and $A(\phi)$ are in principle arbitrary, but they are not all independent. For example, field redefinitions allow us to set either $f_0(\phi) = 1$ or $A(\phi) = 1$, which corresponds to working in the so-called “Einstein” or “Jordan” frames, respectively. This Lagrangian encompasses models in which gravity is coupled to a single scalar field ϕ in all possible ways, including all linearly independent quadratic curvature corrections to GR.

Scalar-tensor gravity with generic coupling, sometimes called Bergmann-Wagoner theory [24, 25], corresponds to setting $f_1(\phi) = f_2(\phi) = 0$ in Eq. (1). This is one of the oldest and best-studied modifications of GR. If we further specialize to the case where $A(\phi) = 1$, $f_0(\phi) = \phi$, $\varpi(\phi) = \omega_{\text{BD}}/\phi$ and $M(\phi) = 0$ we recover the “standard” Brans-Dicke theory of gravity in the Jordan frame [26]; the Einstein frame corresponds to setting $f_0(\phi) = 1$ instead. In a Taylor expansion of $M(\phi)$, the term quadratic in ϕ introduces a nonzero mass for the scalar (see e.g. [27]). GR is recovered in the limit $\omega_{\text{BD}} \rightarrow \infty$.

Initially motivated by attempts to incorporate Mach’s principle into GR, scalar-tensor theories have remained popular both because of their relative simplicity, and because scalar fields are the simplest prototype of the additional degrees of freedom predicted by most unification attempts [28]. Bergmann-Wagoner theories are less well studied than one might expect, given their long history³. These theories can be seen as the low-energy limit of several proposed attempts to unify gravity with the other interactions or, more pragmatically, as mathematically consistent alternatives to GR that can be used to understand which features of the theory are well-tested, and which features need to be tested in more detail [30]. Most importantly, they meet all of the basic requirements of “serious” contenders to GR, as defined above. They are well-posed and amenable to numerical evolutions [31], and in fact numerical evolutions of binary mergers in scalar-tensor theories have already been performed for both BH-BH [32] and NS-NS [33] binaries. At present, the most stringent bound on the coupling parameter of standard Brans-Dicke theory ($\omega_{\text{BD}} > 40,000$) comes from Cassini measurements of the Shapiro time delay [1], but binary pulsar data are rapidly becoming competitive with the Cassini bound: observations of binary systems containing at least one pulsar, such as the pulsar-white dwarf binary PSR J1738+0333, already provide very stringent bounds on Bergmann-Wagoner theories [12].

The third line of the Lagrangian (1) describes theories quadratic in the curvature. The requirement that the field equations should be of second order means that corrections quadratic in the curvature must appear in the Gauss-Bonnet (GB) combination $\mathcal{R}_{\text{GB}}^2 = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$. We also allow for a dynamical Chern-Simons correction proportional to the wedge product

³ Tensor multi-scalar theories of gravity have also been investigated in depth (see e.g. [29] and references therein), but we will not consider them here.

$R_{abcd}^* R^{abcd}$ [34]. Following [35], we will call these models “extended scalar-tensor theories”. These theories have been extensively investigated from a phenomenological point of view: the literature includes studies of Solar-system tests [36, 37], BH solutions and dynamics [38, 39, 40, 41, 42], NS structure [43, 44, 35] and binary dynamics [45, 46, 47]. While the interest of this class of theories is undeniable, and recent work has highlighted very interesting phenomenological consequences for the dynamics of compact objects, it is presently unclear whether they admit a well defined initial value problem and whether they are amenable to numerical evolutions. In analytical treatments these theories are generally regarded as “effective” rather than fundamental (see e.g. [45] for a discussion), and treated in a small-coupling approximation that simplifies the field equations and ensures that the field equations are of second order.

The Lagrangian (1) is more generic than it may seem. For example, it describes – at least at the formal level – theories that replace the Ricci scalar R by a generic function $f(R)$ in the Einstein-Hilbert action, because these theories can always be cast as (rather anomalous) scalar-tensor theories via appropriate variable redefinitions [18, 48]. Unfortunately the mapping between $f(R)$ theories and scalar-tensor theories is in general multivalued, and one should be very careful when considering the scalar-tensor “equivalent” of an $f(R)$ theory (see e.g. [49]). Recently popular theories that are not encompassed by the Lagrangian above include e.g. Einstein-aether theory [50], Hořava gravity [51], Bekenstein’s TeVeS [52], massive gravity theories [53] and “Eddington inspired gravity” [54], which is equivalent to GR in vacuum, but differs from it in the coupling with matter.

An overview of these theories is clearly beyond the scope of this paper. From now on I will focus on the surprisingly overlooked fact that theories of the Bergmann-Wagoner type, which are among the simplest options to modify GR, allow us to introduce very interesting dynamics by simply giving a nonzero mass to the scalar field. Scalar fields predicted in unification attempts are generally massive, so this “requirement” is in fact very natural. I will now argue that massive scalar fields give rise to extremely interesting phenomena in BH physics (Section 3) and binary dynamics (Section 4).

3 Black hole dynamics and superradiance

With the caveat that measurements based on the Kerr *metric* alone do not necessarily differentiate between GR and alternative theories of gravity, BHs are ideal astrophysical laboratories for strong field gravity. Recent results in numerical relativity (see e.g. [55, 56]) confirmed that the dynamics of BHs can be approximated surprisingly well using linear perturbation theory (see Chandrasekhar’s classic monograph [57] for a review). In perturbation theory, the behavior of test fields of any spin (e.g. $s = 0, 1, 2$ for scalar, electromagnetic and gravitational fields) can be described in terms of an effective potential [57, 58]. For massless scalar perturbations of a Kerr BH, the potential is such that: 1) it goes to zero at the BH horizon, which (introducing an appropriate

radial “tortoise coordinate” r_* [57]) corresponds to $r_* \rightarrow -\infty$; 2) it has a local maximum located (roughly) at the light ring; 3) it tends to zero as $r_* \rightarrow \infty$. A nonzero scalar mass does not qualitatively alter features 1) and 2), but it creates a *nonzero potential barrier* such that $V \rightarrow m^2$ (where m is the mass of the field in natural units $G = c = \hbar = 1$) at infinity. Because of the nonzero potential barrier, the potential can now accommodate *quasibound states* in the potential well located between the light-ring maximum and the potential barrier at infinity (cf. e.g. Fig. 7 of [59]). These states are quasibound because the system is dissipative. In fact, under appropriate conditions the system can actually be unstable. The stable or unstable nature of BH perturbations is determined by the shape of the potential and by a well-known feature of rotating BHs: the possibility of *superradiant amplification* of perturbation modes. I will begin by discussing stable perturbations in Section 3.1, and then I will turn to superradiantly unstable configurations in Section 3.2.

3.1 Stable dynamics in GR: quasinormal modes

Massless (scalar, electromagnetic or gravitational) perturbations of a Kerr BH have a “natural” set of boundary conditions: we must impose that waves can only be ingoing at the horizon (which is a one-way membrane) and outgoing at infinity, where the observer is located. Imposing these boundary conditions gives rise to an eigenvalue problem with complex eigenfrequencies, that correspond to the so-called BH quasinormal modes [58]. The nonzero imaginary part of the modes is due to damping (radiation leaves the system both at the horizon and at infinity), and its inverse corresponds to the damping time of the perturbation. By analogy with damped oscillations of a ringing bell, the gravitational radiation emitted in these modes is often called “ringdown”.

The direct detection of ringdown frequencies from perturbed BHs will provide stringent *internal* tests that astrophysical BHs are indeed described by the Kerr solution. The possibility to carry out such a test depends on the signal-to-noise ratio (SNR) of the observed GWs: typically, SNRs larger than ~ 30 should be sufficient to test the Kerr nature of the remnant [60]. While these tests may be possible using Earth-based detectors, they will probably require observations of relatively massive BH mergers with total mass $\sim 10^2 M_\odot$. A detection of such high-mass mergers would be a great discovery in and by itself, given the dubious observational evidence for intermediate-mass BHs [61]. On the other hand, the existence of massive BHs with $M \gtrsim 10^5 M_\odot$ is well established, and space-based detectors such as (e)LISA [62, 63, 64] have a formidable potential for observing the mergers of the lightest supermassive BHs with large SNR throughout the Universe (see e.g. Fig. 16 of [63]). Any such observation would yield stringent “internal” strong-field tests of GR. Furthermore, ringdown observations can be used to provide extremely precise measurements of the remnant spins [60]. Since the statistical distribution of BH spins encodes information on the past history of assembly and growth of the massive BH population in the Universe [65], spin measurements can be used to discrim-

inate between astrophysical models that make different assumptions on the birth and growth mechanism of massive BHs [66,67].

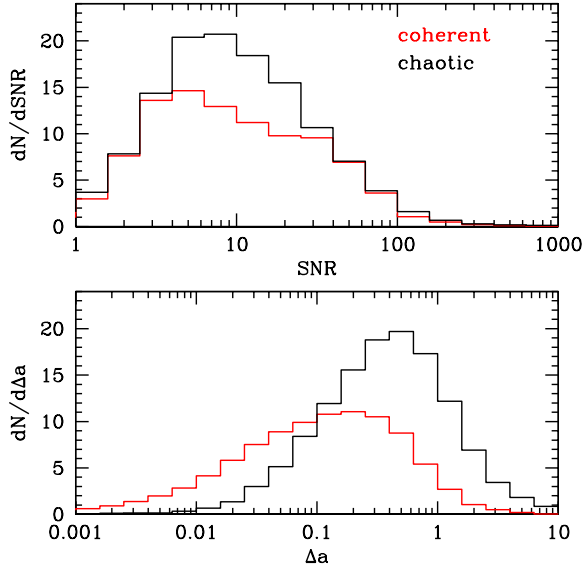


Fig. 1 SNR distribution of detected events (top histogram) and remnant spin measurement accuracy for hierarchical BH formation models with massive seeds and either coherent (red) or chaotic (black) accretion: cf. [68, 65,67] for further details. (Figure courtesy of A. Sesana.)

The potential of a space-based mission like (e)LISA to perform “internal” tests of GR and constrain the merger history of massive BHs using ringdown observations is illustrated in Fig. 1. There we consider hypothetical (e)LISA detections of ringdown waves (computed using analytic prescriptions from [60]) within two different models for supermassive BH formation. Both models assume a hierarchical evolution starting from heavy BH seeds, but they differ in their prescription for the accretion mode, which is either coherent (leading on average to large spins) or chaotic (leading on average to small spins): see the LISA Parameter Estimation Taskforce study [65] for more details. The histograms show the distribution of SNR and spin measurement accuracy during the two-year nominal lifetime of the eLISA mission [63,64]. Independently of the accretion mode, both models predict that 1) more than ten events would have SNR larger than 30, and 2) a few tens of events would allow ringdown-based measurements of the remnant spin to an accuracy better than $\sim 10\%$. Space-based detectors with six links may identify electromagnetic counterparts to some of these merger events and determine their distance [68,63,64]. While extremely promising, this simple assessment of the potential of ringdown waves to test GR should still be viewed as somewhat pessimistic, because a statistical ensemble of events can bring significantly improvements over indi-

vidual observations: see e.g. [69] for a discussion of this point in the context of graviton-mass bounds with (e)LISA observations of inspiralling BH binaries⁴.

3.2 Unstable dynamics in the presence of massive bosons: superradiant instabilities

As anticipated at the beginning of this section, the existence of a local minimum in the potential for massive scalar perturbations allows for the existence of quasibound states. Detweiler [72] computed analytically the frequencies of these quasibound states, finding that they can induce an instability in Kerr BHs. The physical origin of the instability is BH superradiance, as first pointed out by Press and Teukolsky [73] (see also [74, 75, 76]): scalar waves incident on a rotating BH with frequency $0 < \omega < m\Omega_H$ (where Ω_H is the angular frequency of the horizon) extract rotational energy from the BH and are reflected to infinity with an amplitude which is *larger* than the incident amplitude. The barrier at infinity acts as a reflecting mirror, so the wave is reflected and amplified again. The extraction of rotational energy and the amplification of the wave at each subsequent reflection trigger what Press and Teukolsky called the “black-hole bomb” instability.

Scalar fields. For scalar fields, results by Detweiler and others [72, 73, 77, 78, 74, 79, 80] show that the strenght of the instability is regulated by the dimensionless parameter $M\mu$ (in units $G = c = 1$), where M is the BH mass and $m = \mu\hbar$ is the field mass, and it is strongest when the BH is maximally spinning and $M\mu \sim 1$ (cf. [79]). For a solar mass BH and a field of mass $m \sim 1$ eV the parameter $M\mu \sim 10^{10} \gg 1$, and the instability is exponentially suppressed [78]. Therefore in many cases of astrophysical interest the instability timescale must be larger than the age of the Universe. Strong, astrophysically relevant superradiant instabilities with $M\mu \sim 1$ can occur either for light primordial BHs which may have been produced in the early Universe, or for ultralight exotic particles found in some extensions of the standard model. An example is the “string axiverse” scenario [81, 59], according to which massive scalar fields with 10^{-33} eV $< m < 10^{-18}$ eV could play a key role in cosmological models. Superradiant instabilities may allow us to probe the existence of such ultralight bosonic fields by producing gaps in the BH Regge plane [81, 59] (i.e. the mass/spin plane), by modifying the inspiral dynamics of compact binaries [82, 83, 27] or by inducing a “bosonova”, i.e. collapse of the axion cloud (see e.g. [84, 85, 86]).

⁴ The strength of these tests will depend on two key elements: (i) the *signal-to-noise ratio* (SNR) of individual observations [70], that also affects accuracy in binary parameter estimation, and (ii) the *number N of observations* that can be used to constrain GR. The reason is that, given a theory whose deviations from GR can be parametrized by one or more universal parameters (e.g. coupling constants), the bounds on these parameters will scale roughly with \sqrt{N} . As a matter of fact, the bounds could improve faster than \sqrt{N} if some events are particularly loud: see e.g. [71, 69] for detailed analyses addressing specific modifications to GR in the Advanced LIGO/eLISA context, respectively.

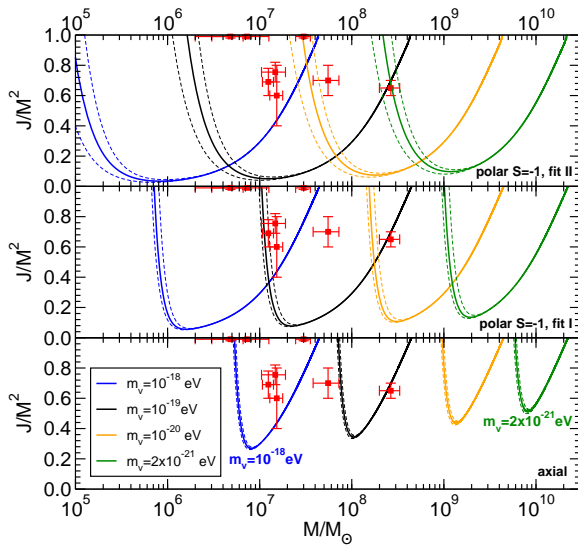


Fig. 2 Contour plots in the BH Regge plane [59] corresponding to an instability timescale shorter than a typical accretion timescale, $\tau_{\text{Salpeter}} = 4.5 \times 10^7$ yr, for different values of the vector field mass $m_v = \mu\hbar$ (from left to right: $m_v = 10^{-18}\text{eV}$, 10^{-19}eV , 10^{-20}eV , $2 \times 10^{-21}\text{eV}$). For polar modes we consider the $S = -1$ polarization, which provides the strongest instability, and we use two different fits to our numerical results. Dashed lines bracket our estimated numerical errors. The experimental points (with error bars) refer to the mass and spin estimates of supermassive BHs listed in Table 2 of [87]; the rightmost point corresponds to the supermassive BH in Fairall 9 [88]. Supermassive BHs lying above each of these curves would be unstable on an observable timescale, and therefore they exclude a whole range of Proca field masses.

Vector fields. It has long been believed that the “BH bomb” instability should operate for all bosonic field perturbations in the Kerr spacetime, and in particular for massive spin-one (Proca) bosons⁵ [79]. A proof of this conjecture was lacking until recently because of technical difficulties in separating the perturbation equations for massive spin-one (Proca) fields in the Kerr background. Pani et al. recently circumvented the problem using a slow-rotation expansion pushed to second order in rotation [91, 92]. The Proca superradiant instability turns out to be stronger than the massive scalar field instability. Furthermore the Proca mass range where the instability would be active is very interesting from an experimental point of view: indeed, as shown in [91], astrophysical BH spin measurements are already setting the most stringent upper bound on the mass of spin-one fields. This can be seen in Fig. 2, which shows exclusion regions in the “BH Regge plane” (cf. Fig. 3 of [59]) obtained

⁵ Superradiant amplification is not possible for fermionic fields: see e.g. [89, 90].

by setting the instability timescale equal to the (Salpeter) accretion timescale $\tau_{\text{Salpeter}} = 4.5 \times 10^7$ yr. The idea here is that a conservative bound on the critical mass of the Proca field corresponds to the case where the instability spins BHs down *faster* than accretion could possibly spin them up. Instability windows are shown for four different masses of the Proca field ($m_v = 10^{-18}$ eV, 10^{-19} eV, 10^{-20} eV and 2×10^{-21} eV) and for two different classes of unstable Proca modes: “axial” modes (bottom panel) and “polar” modes with polarization index $S = -1$, which provides the strongest instability (top and middle panels). All regions above the instability window are ruled out⁶. The plot shows that essentially *any* spin measurement for supermassive BHs with $10^6 M_\odot \lesssim M \lesssim 10^9 M_\odot$ would exclude a wide range of vector field masses [91, 92]. Massive vector instabilities do not – strictly speaking – provide “external” tests of GR, but rather tests of perturbative dynamics within GR; quite interestingly, they provide constraints on possible mechanisms to generate massive “hidden” U(1) vector fields, which are predicted by various extensions of the Standard Model [93, 94, 95, 96]. The results discussed in this section are quite remarkable, because they show that astrophysical measurements of nonzero spins for supermassive BHs can already place the strongest constraints on the mass of hypothetical vector bosons (for comparison, the Particle Data Group quotes an upper limit $m < 10^{-18}$ eV on the mass of the photon [97]).

4 Present and future tests of massive scalar-tensor theories

So far I discussed “internal” tests of GR from future GW observations of stable BH dynamics (ringdown waves). I also summarized how superradiant instabilities can be used to place bounds on the masses of scalar and vector fields, which emerge quite naturally in extensions of the Standard Model [81, 59, 93, 94, 95, 96]. In this Section I address a slightly different but related question, namely: what constraints on the mass and coupling of scalar fields are imposed by Solar System observations? Shall we be able to constrain these models better (or prove that scalar fields are indeed needed for a correct description of gravity) using future GW observations?

4.1 Solar System bounds

In [27] we investigated observational bounds on massive scalar-tensor theories of the Brans-Dicke type. In addition to deriving the orbital period derivative due to gravitational radiation, we also revisited the calculations of the Shapiro time delay and of the Nordtvedt effect in these theories (cf. [1] for a detailed and updated treatment of these tests).

⁶ While our numerical results for the axial modes are supported by an analytical formula, in the polar case we have used two different functions to fit the numerical data at second order in the BH spin.

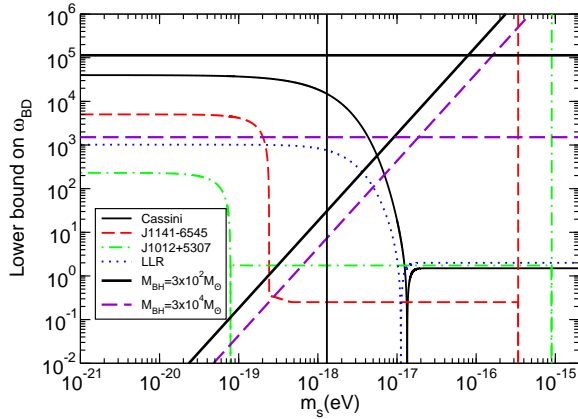


Fig. 3 Lower bound on $(\omega_{\text{BD}} + 3/2)$ as a function of the mass of the scalar m_s from the Cassini mission data (black solid line; cf. [98]), period derivative observations of PSR J1141-6545 (dashed red line) and PSR J1012+5307 (dot-dashed green line), and Lunar Laser Ranging experiments (dotted blue line). Vertical lines indicate the masses corresponding to the typical radii of the systems: 1AU (black solid line) and the orbital radii of the two binaries (dashed red and dot-dashed green lines). Note that the theoretical bound on the coupling parameter is $\omega_{\text{BD}} > -3/2$.

The comparison of our results for the orbital period derivative, Shapiro time delay and Nordtvedt parameter against recent observational data allows us to put constraints on the parameters of the theory: the scalar mass m_s and the Brans-Dicke coupling parameter ω_{BD} . These bounds are summarized in Figure 3. We find that the most stringent bounds come from the observations of the Shapiro time delay in the Solar System provided by the Cassini mission (which had already been studied in [98]). From the Cassini observations we obtain $\omega_{\text{BD}} > 40,000$ for $m_s < 2.5 \times 10^{-20} \text{eV}$, while observations of the Nordtvedt effect using the Lunar Laser Ranging (LLR) experiment yield a slightly weaker bound of $\omega_{\text{BD}} > 1,000$ for $m_s < 2.5 \times 10^{-20} \text{eV}$. Possibly our most interesting result concerns observations of the orbital period derivative of the circular white-dwarf neutron-star binary system PSR J1012+5307, which yield $\omega_{\text{BD}} > 1,250$ for $m_s < 10^{-20} \text{eV}$. The limiting factor here is our ability to obtain precise measurements of the masses of the component stars as well as of the orbital period derivative, once kinematic corrections have been accounted for. However, there is considerably more promise in the eccentric binary PSR J1141-6545, a system for which remarkably precise measurements of the orbital period derivative, the component star masses and the periastron shift are available. The calculation in [27] was limited to circular binaries, and we are currently working to generalize our treatment to eccentric binaries in order to carry out a more meaningful and precise comparison with observations of PSR J1141-6545.

4.2 Gravitational-wave tests

Binary pulsar observations can test certain aspects of strong-field modifications to GR, such as the “spontaneous scalarization” phenomenon in scalar-tensor theories [6], and interesting tests are also possible with current astronomical observations [3]. However a real breakthrough is expected to occur in the near future with the *direct* detection of GWs from the merger of compact binaries composed of BHs and/or NSs. One of the most exciting prospects of the future network of GW detectors (Advanced LIGO/Virgo [99], LIGO-India [100] and KAGRA [101] in the near future; third-generation Earth-based interferometers like the Einstein Telescope [102] and a space-based, LISA-like mission [62, 63, 64] in the long term) is precisely their potential to test GR in strong-field, high-velocity regimes inaccessible to Solar System and binary pulsar experiments. Second-generation interferometers such as Advanced LIGO should detect a large number of compact binary coalescence events [103, 104]. Unfortunately from the point of view of testing GR, most binary mergers detected by Advanced LIGO/Virgo are expected to have low signal-to-noise ratios (a possible exception being the observation of intermediate-mass BH mergers [105], that would be a great discovery in and by itself). Third-generation detectors such as the Einstein Telescope will perform significantly better in terms of parameter estimation and tests of alternative theories [106, 107]. Here I will argue (using the example of massive scalar-tensor theories) that an (e)LISA-like mission will be an ideal instrument to test GR [63, 64] by providing two examples: (1) bounds on massive scalar-tensor theories using (e)LISA observations of intermediate mass-ratio inspirals, and (2) the possibility to observe an exotic phenomenon related once again to superradiance, i.e., floating orbits.

Bounds on massive scalar-tensor theories from intermediate mass-ratio inspirals. In general, the gravitational radiation from a binary in massive scalar-tensor theories depends on both the scalar field mass m_s and the coupling constant ω_{BD} [108, 27]. If the field is massless, corrections to the GW phasing are proportional to $1/\omega_{\text{BD}}$, and therefore comparisons of the phasing in GR and in scalar-tensor theories yield bounds on ω_{BD} [109, 108]. By computing the GW phase in the stationary-phase approximation, one finds that the scalar mass always contributes to the phase in the combination m_s^2/ω_{BD} , so that GW observations of nonspinning, quasicircular inspirals can only set upper limits on $m_s/\sqrt{\omega_{\text{BD}}}$ [110]. For large SNR ρ , the constraint is inversely proportional to ρ . The order of magnitude of the achievable bounds is essentially set by the lowest frequency accessible to the GW detector, and it can be understood by noting that the scalar mass and GW frequency are related (on dimensional grounds) by $m_s(\text{eV}) = 6.6 \times 10^{-16} f(\text{Hz})$, or equivalently $f(\text{Hz}) = 1.5 \times 10^{15} m_s(\text{eV})$. For eLISA, the lower cutoff frequency (imposed by acceleration noise) $f_{\text{cut}} \sim 10^{-5}$ Hz corresponds to a scalar of mass $m_s \simeq 6.6 \times 10^{-21}$ eV. For Earth-based detectors the typical seismic cutoff frequency is $f_{\text{cut}} \sim 10$ Hz, corresponding to $m_s \sim 6.6 \times 10^{-15}$ eV. This simple argument shows that space-based detectors can set $\sim 10^6$ stronger bounds on the scalar mass than Earth-based detectors.

An explicit calculation shows that the best bounds are obtained from (e)LISA observations of the intermediate mass-ratio inspiral of a neutron star into a BH of mass $M_{\text{BH}} \lesssim 10^3 M_\odot$, and that they would be of the order

$$\left(\frac{m_s}{\sqrt{\omega_{\text{BD}}}} \right) \left(\frac{\rho}{10} \right) \lesssim 10^{-19} \text{ eV}. \quad (2)$$

In summary, GW observations will provide two constraints: a lower limit on ω_{BD} (corresponding to horizontal lines in Fig. 3) and an upper limit on $m_s/\sqrt{\omega_{\text{BD}}}$ (corresponding to the straight diagonal lines in Fig. 3). Therefore GW observations would exclude the complement of a trapezoidal region on the top left of Fig. 3. Straight (dashed) lines show the bounds from eLISA observations of NS-BH binaries with SNR $\rho = 10$ when the BH has mass $M_{\text{BH}} = 300 M_\odot$ ($M_{\text{BH}} = 3 \times 10^4 M_\odot$, respectively). The plot shows that GW observations with $\rho = 10$ become competitive with binary pulsar bounds when $m_s \gtrsim 10^{-19}$ eV, and competitive with Cassini bounds when $m_s \gtrsim 10^{-18}$ eV, with the exact “transition point” depending on the SNR of the observation (for a GW observation with SNR $\rho = 100$ the “straight line” bounds in Fig. 3 would be ten times higher). Therefore in this particular theory a single high-SNR observation (or the statistical combination of several observations, see e.g. [69]) may yield better bounds on the scalar coupling than weak-gravity observations in the Solar System when $m_s \gtrsim 10^{-18}$ eV.

Floating orbits. It is generally expected that small bodies orbiting around a BH will lose energy in gravitational waves, slowly inspiralling into the BH. In [82] we showed that the coupling of a massive scalar field to matter leads to a surprising effect: because of superradiance, orbiting objects can hover into “floating orbits” for which the net gravitational energy loss at infinity is entirely provided by the BH’s rotational energy. The idea is that a compact object around a rotating BH can excite superradiant modes to appreciable amplitudes when the frequency of the orbit matches the frequency of the unstable quasibound state. This follows from energy balance: if the orbital energy of the particle is E_p , and the total (gravitational plus scalar) energy flux is $\dot{E}_T = \dot{E}^g + \dot{E}^s$, then

$$\dot{E}_p + \dot{E}^g + \dot{E}^s = 0. \quad (3)$$

Usually $\dot{E}^g + \dot{E}^s > 0$, and therefore the orbit shrinks with time. However it is possible that, due to superradiance, $\dot{E}^g + \dot{E}^s = 0$. In this case $\dot{E}_p = 0$, and the orbiting body can “float” rather than spiralling in [111, 73]. The system is essentially a “BH laser”, where the orbiting compact object is producing stimulated emission of *gravitational* radiation: because the massive scalar field acts as a mirror, negative scalar radiation ($\dot{E}^s < 0$) is dumped into the horizon, while gravitational radiation can be detected at infinity. Orbiting bodies remain floating until they extract sufficient angular momentum from the BH, or until perturbations or nonlinear effects disrupt the orbit. For slowly rotating and nonrotating BHs floating orbits are unlikely to exist, but resonances at orbital frequencies corresponding to quasibound states of the scalar field can speed up the inspiral, so that the orbiting body “sinks”. A detector like

(e)LISA could easily observe these effects [82,83], that would be spectacular smoking guns of deviations from general relativity.

5 Conclusions

The three examples discussed in this paper (ringdown tests of the BH no-hair theorem, bosonic superradiant instabilities in rotating BHs and GW tests of massive scalar-tensor theories) illustrate that astrophysical BHs, either in isolation or in compact binaries, can be spectacular nature-given laboratories for fundamental physics. We can already use astrophysical observations to do fundamental physics (e.g. by setting bounds on the masses of scalar and vector fields using supermassive BH spin measurements), but the real goldmine for the future of “fundamental astrophysics” will be GW observations. In order to fully realize the promise of GWs as probes of strong-field gravity we will need several detections with large SNR. Second- and third-generation Earth-based interferometers will certainly deliver interesting science, but a full realization of strong-field tests and fundamental physics with GW observations may have to wait for space-based GW detectors. We’d better make sure they happen in our lifetime.

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